AD-A124 591 AN ANALYTICAL APPROACH FOR ASSESSING VULNERABILITY TO COUNTERFIRE(U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA K F SNIDER OCT 82 1/1

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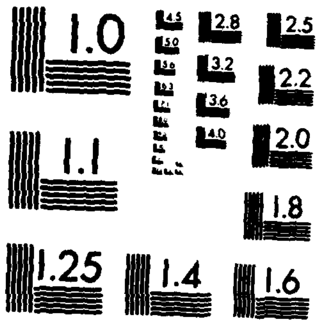

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NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

AN ANALYTICAL APPROACH FOR ASSESSING
VULNERABILITY TO COUNTERFIRE

by

Keith F. Snider

Advisors: J. G. Taylor and S. J. Paek

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A124 591	3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
An Analytical Approach for Assessing Vulnerability to Counterfire

5. TYPE OF REPORT & PERIOD COVERED
Master's Thesis

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
Keith F. Snider

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Naval Postgraduate School
Monterey, California 93940

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS
Naval Postgraduate School
Monterey, California 93940

12. REPORT DATE
October 1982

13. NUMBER OF PAGES
51

14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Counterfire
Sound Ranging
Artillery Locating Radars
Artillery Movement

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
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20. Abstract (Continued)

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An Analytical Approach for Assessing Vulnerability to Counterfire

by

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
October, 1982

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ABSTRACT

An analytical approach for assessing the vulnerability of an artillery battery in terms of the probability that the battery receives counterfire is developed. This approach is based on a simplified model that estimates the probability of detection by indirect fire weapons locating systems of artillery weapons firings, and on recent work by the Mitre Corporation in determining probabilities of counterfire on the basis of battery exposure times and enemy counterfire response times. An illustrative example is given to demonstrate the approach, and parametric variations are performed to investigate the impact of changing tactics and weapons characteristics.

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I. INTRODUCTION

A. PURPOSE

The purpose of this thesis is to examine some approaches for investigating the vulnerability of a field artillery battery to counterfire. The motivation for the analysis stems from recent technological advances in indirect fire weapons locating systems (WLS). The capabilities of these new systems and the potential for further advances in the area point out the need for continual analysis of the survivability aspects of field artillery tactics and doctrine.

B. OVERVIEW

A brief discussion of the nature of the problem and a description of the counterfire system are presented in this chapter. Chapter II discusses the derivation of a simplified expression for the probability of detection of a firing artillery battery by an array of WLS assets. Chapter III examines recent work by the Mitre Corporation in the determination of the probability of a battery receiving counterfire given that the battery has been detected and located. Chapter IV discusses an expression for the probability of counterfire obtained from the methodologies of Chapters II and III. An illustrative example is given to demonstrate the working of the model. Conclusions are presented in Chapter V.

NATURE OF THE PROBLEM

The field artillery commander must be concerned with several threats to the survivability of his firing units. Probably the most serious threats are hostile ground attacks, air attacks, and counterbattery fires. Of these, the one to which his units have few or no means to respond is counterbattery fire. His troops may return fire on attacking enemy aircraft, or attempt to repel a ground attack with their organic weapons, but they will usually have no warning of an impending counter-battery attack, nor will they usually have any indication of the location of the hostile artillery unit conducting the attack.

The battery commander can minimize the possibilities of attacks on his unit by careful positioning, camouflage, the use of local security elements, and proper communications procedures. Such steps will, to the greatest extent possible, prevent visual detection and detection by communications direction finding systems, even while the battery is firing.

However, there exists other means by which artillery weapons may be located. Two of these which have as their primary function the detection and location of firing weapons are weapons locating radars and sounding ranging systems. (Flash systems, which are normally closely associated with sound ranging systems, depend on visual means for detecting artillery weapons and so will not be addressed.) These two systems make use of the actual weapons firings to determine the locations of these weapons. Because a battery must provide supporting fires to the maneuver forces, it is especially susceptible to detection by

these two means, regardless of any steps the commander may take to conceal his unit.

Three alternatives are normally suggested to reduce the possibilities of casualties due to counterfire: dispersion of weapons, "hardening" of individual weapons positions, and displacement of the battery to a new position [Ref. 1]. Dispersion is normally practiced, and positions may be hardened when the situation permits, but a major advantage of movement as an alternative in any combat situation is that it is the only method of the three which effectively eliminates the possibilities of casualties due to counterfire once the position is vacated. The necessity for frequent moves is generally recognized because of the anticipated fast-moving pace of modern combat and because the longer a battery remains in a position, the greater its risk becomes of being located by all enemy detection means [Ref. 1].

The most significant problem with displacing artillery batteries frequently to avoid the effects of counterfire is that a battery which is moving cannot provide immediate fire support [Ref. 1]. Other problems include increased risks of visual detection, coordination problems with other friendly units, troop fatigue, and increased wear and tear of equipment.

A battery may be required to move under several circumstances. It must move whenever it cannot provide fire support to the maneuver units. It may move when its position becomes untenable due to an actual or impending enemy attack [Ref. 1]. Intelligence sources and security elements may provide the commander with some warning of ground or air

strikes, but he usually will have little or no information on whether or not his unit has been detected and targeted for a counterbattery attack. If this information was available to him and the tactical situation permitted, he could move those vulnerable units before the counterfire missions were fired, thereby reducing the possibilities of troop and equipment losses.

It is unrealistic, given current capabilities and the inherently uncertain nature of combat, to expect that this type of precise information could be made available to the commander in a timely manner. However, it is possible to develop methodologies which can lead to a better understanding of the problem. To this end, approaches of the type developed in subsequent chapters are suggested as means by which such information can be obtained to serve as a possible basis for decisions concerning movement of artillery units.

D. THE COUNTERFIRE SYSTEM

The counterfire system refers to those capabilities which enable a force to detect, locate and engage the fire support assets of another force. It includes target acquisition elements, intelligence and coordination elements, and firing elements. A brief discussion follows of the two target acquisition systems pertinent to this thesis, weapons locating radars and sound ranging, as well as a discussion of the functions of the intelligence and coordination element, to which will be referred generically as the counterfire element.

1. Weapons Locating Radars

Weapons locating radars have existed in the counterfire system for several years, but only recently have they been improved to the extent that they have become the most accurate means for locating artillery weapons other than by direct observation. A variety of radars are in use or in production, but the principles behind the functioning of each are basically the same. The radar "scans" a sector of the battlefield where it is believed enemy artillery is located. The path of flight of a projectile can be determined on the basis of tracking by the radar or by the intersection by the projectile of multiple radar beams [Ref. 2]. The path may then be extrapolated back to the position from which the projectile was fired. The effectiveness of radars may be degraded by precipitation and multiple weapons firings within a short period. A significant disadvantage is that, because radars are active emitters, they are subject to electronic countermeasures.

2. Sound Ranging

Sound ranging as a means for locating artillery weapons came into widespread use during World War II. Weapons are located on the basis of the relative times of arrival of the sound waves caused by their firings at a "base" of several accurately located microphones placed roughly perpendicular to the anticipated direction of fire. Meteorological conditions which may affect the propagation of the sound waves are taken into account. A major advantage of this means is that it is passive in nature and therefore is less susceptible to electronic countermeasures. The accuracy of the system in determining locations of

artillery weapons may be degraded by very windy conditions or by hilly terrain. The occurrence of many weapons firings in a short time may also significantly degrade the performance of the system [Ref. 3]. In addition, preparations for the emplacement of a sound base may be extensive in terms of required survey and communications support.

3. The Counterfire Element

The counterfire element, usually found at the headquarters of the unit controlling counterfire capabilities, acts as an intelligence and coordination agency to determine hostile weapons locations and dispositions and to direct the engagement of those weapons. Information from all sources is collected in an attempt to put together as accurate a picture of the battle as possible.

In response to guidance from the commander, a series of engagement rules, called attack guidance, are established for the conduct of counterfire operations [Ref. 4]. For example, he may specify that no target be engaged with counterfire unless it is located to within two hundred meters, or he may specify that all artillery weapons which are located are to be attacked with a minimum of two battalion volleys. These rules serve to outline the types of targets to be attacked, the method of attack, and the conditions under which they are to be engaged.

Upon receipt of a piece of combat intelligence, the counterfire element determines if a target location can be produced which is consistent with the existing attack guidance. (The intelligence may already be in the form of a target location, especially if it was received from a WLS.) The relative priorty of the target and the availability of resources are determined, and if the decision is made to

engage the target, efforts are made to assess the effectiveness of the attack.

In some instances, a WLS may be attached directly to an artillery firing unit. In this case, the WLS would report target information to that unit's fire direction center (FDC). The FDC may act upon the information by firing a counterfire mission, or it may simply pass on the information to the counterfire element for further processing [Ref 5]. In either case, the decision process through which the FDC would go is fundamentally the same as that of the counterfire element.

The relevance of a discussion of the counterfire element to this thesis may be obvious when one considers the possibilities of cancelled counterfire missions due to priorities and resource constraints, of time delays caused by processing the counterfire missions, or of communications failures between the various elements which make up the counterfire system. The vulnerability of artillery weapons is a function of many variables, a very important one being the organization and efficiency of the enemy counterfire element. An analysis which fails to consider this may result in conclusions which are either optimistic or pessimistic, depending on the viewpoint of the analyst.

E. THE PROCESS BY WHICH AN ARTILLERY BATTERY RECEIVES COUNTERFIRE

The process by which an artillery battery may be engaged with counterfire is described as a series of discrete events as shown in Table 1. Each event has a certain probability of occurrence, and if these events are assumed to be independent, the probability of the firing battery receiving counterfire is then given by the product of the probabilities of these events occuring.

Table 1. Events in the Counterfire Process

1. An artillery weapon fires.
2. The firing is detected by a WLS.
3. The WLS produces a location of the weapon.
4. The target information is transmitted to the counterfire element.
5. The counterfire element, after considering: a. Priorities b. Resources c. Accuracy of information determines that a counterfire mission is to be fired.
6. The counterfire element transmits the counterfire mission to a firing unit.
7. That unit fires the mission.
8. The target receives the counterfire.

The modeler may choose to account in his analysis for any or all of these events; however, for subsequent examples given in this thesis, all probabilities other than those of events 2 and 8 are assumed to be one. Reasons for this assumption are the high priorities placed on the destruction or suppression of hostile indirect fire weapons and the dedication of a significant portion of available fire support assets to the counterfire role [Ref. 6]. The probability of counterfire is then given by the product of the probabilities of events 2 and 8. A discussion of the derivation of these probabilities, as well as an illustrative example, is given in subsequent chapters.

II. PROBABILITY OF DETECTION

It is generally assumed in analysis that artillery weapons locating systems possess estimable probability of detection functions which usually vary with range to the artillery weapon. For example, the Mitre Corporation used the following expression for the probability of detection for sound ranging systems in the Counterfire Campaign Analysis [Ref. 7]:

$$P\ (\text{detection}) = \frac{1}{1 + \exp(5 \log R/\bar{R})}$$

where: R = range from the WLS to the weapon
$\bar{R}$ = 50% detection range of the WLS

Expressions of this type should be viewed, however, as conditional probabilities given that the detection system is operating at the time of weapons firing and is scanning the area where the firing takes place. In other words, these expressions are probabilities of detection given that the artillery weapon is susceptible to detection by a particular WLS. Following this reasoning, the unconditional probability of detection P_u is given by:

$$P_u = P(\text{detection}|\text{WLS is operating and scanning the proper sector}) \times P(\text{WLS is operating}) \times P(\text{WLS is scanning the proper sector})$$

The worth of considering these additional probabilities in the analysis may be recognized when the following factors are considered.

1. Some weapons locating radars have fairly narrow "fields of view" and so will not be able to scan the entire battlefield at one particular time. Thus, a significant proportion of weapons may escape detection by any one of these systems.

2. Radars, which are active emitters, are susceptible to electronic countermeasures. As a result, they will probably not remain in continuous operation through the course of a battle. Rather, they may scan a certain sector for a period of time, cease operations for a while, and then resume scanning in the same or another sector [Ref. 5].

3. Most detection systems have limitations in the number of weapons they can detect or the number of weapons locations they can process in a certain period of time. Therefore, if the combat becomes very intense with a large number of firings in a very short time, some of the firings may not be detected.

As stated previously, WLS have an estimable conditional detection probability function which will be noted by P_c. To obtain an expression for the unconditional detection probability, the probabilities that the system is operating and scanning the proper area must be determined.

The probability that a WLS is operating at a random point in time, denoted by P_o, will be defined simply as the fraction of time that the system is operating. For example, if a radar scans the battlefield for three minutes, is off for two minutes, and then repeats the same cycle, the probability that it is operating at a certain time is .6. P_o may also account for the time that a system has been "saturated" by multiple firings in a short period of time.

The probability that a system is scanning an area where a particular artillery battery is located is given by P_s, the fraction of the area of concern which the WLS may scan at any one time.

$$P_s = \frac{\text{Area of Scan}}{\text{Total Area of Operation}}$$

(When the total area of operation is less than the area of scan, P_s=1).

P_s may be modeled in several ways. Figure 1 depicts a notional combat scenario in which P_s is described as a ratio of sectors of a circle. P_s in this instance is simple the ratio of the central angles, or w_1/w_2.

Another method for obtaining an expression for P_s (and the method which is employed in subsequent examples in this thesis) is shown in Figure 2. In this case, the area of concern to the WLS is considered to be the distance W between the boundaries or the extension of the boundaries of the unit it supports. If the WLS was a sound ranging base of a target acquisition battery of a U.S. division, the area of concern to that system would be the distance between the division boundaries. An assumption implicit in this concept is that a WLS will not attempt to detect any weapons firings outside the boundaries of its supported unit.

Let r be the range from the WLS to the artillery weapons in thousands of meters. For the purposes of this analysis, r is considered to be the range from where WLS assets are normally located to the area in which hostile artillery assets are normally located. For example, if a WLS is doctrinally positioned eight to ten kilometers from the FEBA and the enemy traditionally positions his artillery four to eight kilometers

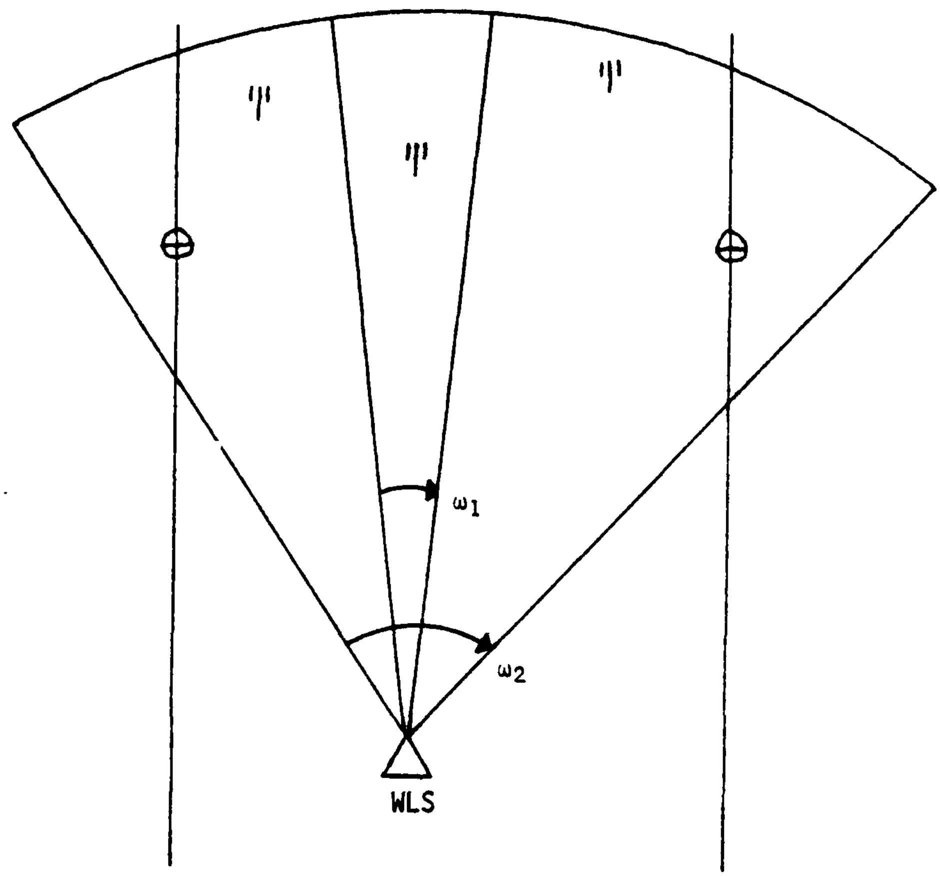

Figure 1. Modeling P_s

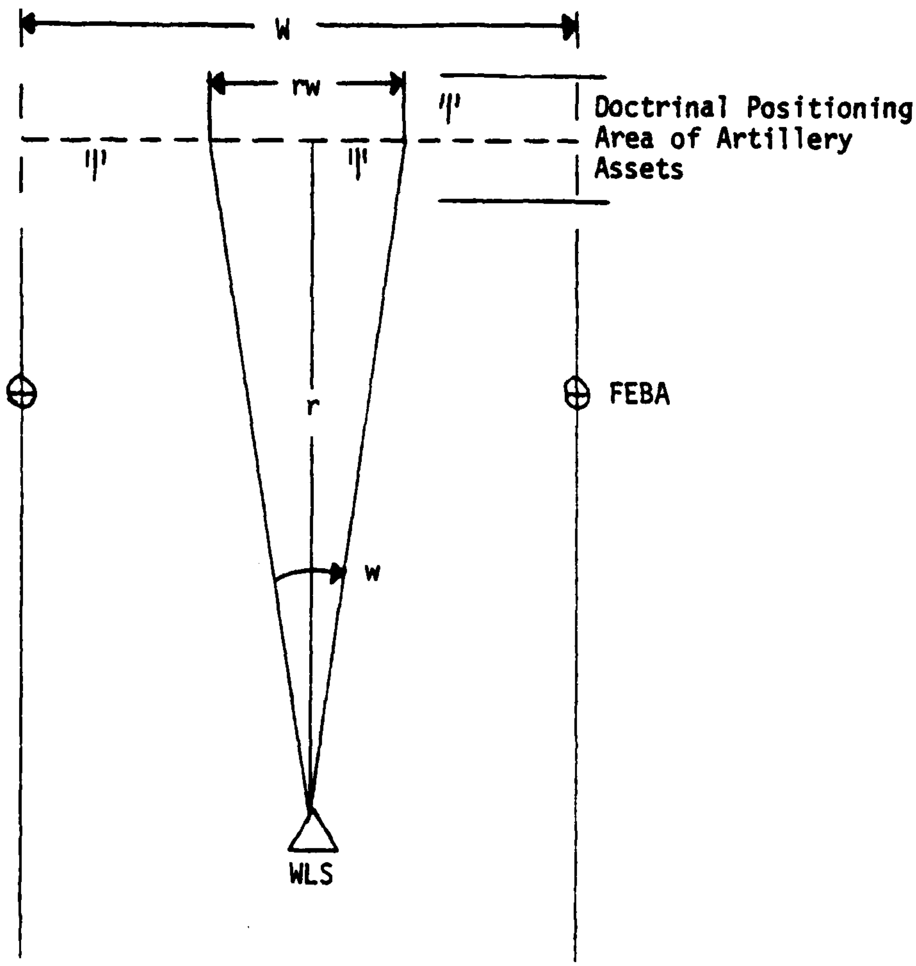

Figure 2. Modeling P_s

behind the FEBA, r would be considered to be fifteen. Let w be the angular distance in mils that the WLS may scan at any one time. Then the width of an area scanned at a distance of r kilometers is rw, and the fraction of the area of concern which is scanned by the WLS is:

$$P_s = \frac{rw}{W} \qquad (1)$$

for the method described by Figure 2.

Regardless of the method for deriving P_s, the following expression will then define the unconditional probability of detection of a particular firing artillery battery by a single WLS:

$$P_u = P_c \; P_o \; P_s \qquad (2)$$

For several WLS, the probability that at least one detects a particular weapons firing is given by:

$$P_d = 1 - \prod_{i=1}^{m} (1 - P_{ui}) \qquad (3)$$

where m = number of WLS

assuming independence between the WLS. This expression is then the probability of occurrence of event 2 of Table 1.

It is recognized that equation (3) represents only a rough approximation of, and because of the assumption of independence between WLS, a "pessimistic" (from the view of the artillery battery) expression for the probability of detection. The assumption of independence may be questioned, and the issue of sector overlap among WLS may be raised. However, accounting for these factors in the methodology would raise

considerably the complexity of the analysis and may require the use of a detailed simulation. The corresponding increase in accuracy may not outweigh a significantly simpler analytical expression. Therefore, equation (3) will be used in subsequent analysis.

III. CONDITIONAL PROBABILITY OF RECEIVING COUNTERFIRE

A. GENERAL

In connection with the Mitre Corporation's Division Support Weapons System (DSWS) Survivability Analysis [Ref. 8], Niedenfuhr presented an elementary quantitative analysis of the effects on DSWS survivability of rates of fire, movement doctrine, and enemy counterfire response times. The specific analysis performed focused only on DSWS; however, the general methodology which Niedenfuhr developed may be applied to other artillery weapons systems as well.

B. METHODOLOGY

Niedenfuhr defines an average period of time a battery is at risk to counterfire in a particular location as the time from the start of the battery's first fire mission to the time when that location is vacated. This period of time, denoted by $\bar{t}_E$, is expressed as follows:

$$\bar{t}_E = \frac{mM}{Rn} + (m-1)t_p + t_d + (m-1)t_w \qquad (4)$$

where:
- m = average number of mission fired from a position
- M = average number of rounds fired per mission
- R = rate of fire of individual howitzers
- n = number of howitzers per firing unit
- t_p = average mission preparation time (time for reaiming weapons)
- t_d = average displacement time
- t_w = average waiting time between missions (includes technical fire direction).

Because of the expected uncertainties of combat, the actual exposure time is taken to be a random variable, T_E, with distribution $P(t_E)$. Due to the nature of the DSWS, Niedenfuhr expects that relatively short exposure times will dominate and assumes that T_E is distributed exponentially with density function:

$$p(t_E) = \frac{1}{\bar{t}_E} \exp\left(\frac{-t_E}{\bar{t}_E}\right)$$

The time at which counterfire is received is given by T_R (enemy response time) which is a function of the intensity of combat, effectiveness of enemy equipment and troops, and enemy counterfire organizations and procedures. T_R is also considered to be a random variable with an exponential distribution with a lower limiting value of t_L, the minimum possible enemy response time. The probability that T_R is less than t_L is zero and, for T_R greater than or equal to t_L, the density function of T_R is given by:

$$p(t_R) = \frac{1}{t} \exp\left(\frac{-(t_R - t_L)}{t}\right)$$

where t is a parameter. The average response time is then given by:

$$\bar{t}_R = t_L + t$$

For the counterfire to be effective, the response time T_R must be less than the exposure time T_E. The probability of this occurring is obtained by:

$$P(T_R < T_E) = \int_{t_L}^{\infty} \frac{1}{\bar{t}_E} \exp\left(\frac{-t_E}{\bar{t}_E}\right)\left[\int_{t_L}^{t_E} \frac{1}{\bar{t}} \exp\left(\frac{-(t_R - t_L)}{\bar{t}}\right) dt_R\right] dt_E \qquad (5)$$

which reduces to

$$P(T_R < T_E) = \left[\frac{\dfrac{\bar{t}_E}{t_L}}{\dfrac{\bar{t}_E}{t_L} + \dfrac{\bar{t}_R}{t_L} - 1}\right] \exp\left(\frac{-t_L}{\bar{t}_E}\right)$$

Niedenfuhr continues his analysis by obtaining expressions for daily survival probabilities based on expected fractional damage values for DSWS units rec iving counterfire. The model is then exercised by varying the parameters of interest to investigate the results of different tactics of movement, rates of fire, and organizations. For a complete discussion of the assumptions, methodologies, results, and conclusions, see Reference 8.

C. DISCUSSION OF THE DSWS MODEL

The analytical results which Niedenfuhr's model produces are useful for developing insights into the dynamics of the counterfire process. The model is enhanced by the explicit treatment of exposure times and response times as random variables in recognition of the stochastic nature of combat. It is interesting to note the parallels between the theory of this model and the theory of stochastic duels, in which the times for duelists (for example, opposing weapons systems) to kill a passive target are considered to be random variables. The prediction of

the outcome of a duel then involves, as in Niedenfuhr's model, the determination of the probability that one random variable is less than another. Taylor gives a brief and excellent treatment of the basic theory of the probability that one random variable is less than another, as well as an application to the theory of stochastic duels [Ref. 9].

Niedenfuhr's approach is useful for developing an expression for the probability of occurrence of event 8 of Table 1; that is, the probability that a battery receives counterfire in a particular position. This probability is conditional, however, given that the battery has been detected and targeted for a counterfire attack. The structure of the methodology and the definition of the variables implicitly assume that a detection occurs on the first mission fired, when in fact the probability that this would occur may be quite small.

In some situations, equation (4) may lead to incorrect exposure times. This equation will only yield correct results when M is some positive integer multiple of n. The term M/n from equation (4) defines the number of volleys fired, and if M is not an integer multiple of n, a "fraction of a volley" will occur. Intuitively, it is obvious that the firing of a fraction of a volley requires exactly the same amount of time as the firing of a full volley, but equation (4) will not reflect this. The problem may be solved quite simply by restating equation (4) as:

$$\bar{t}_E = \frac{m\ Nv}{R} + (m-1)t_p + t_d + (m-1)t_w \qquad (4a)$$

where Nv is the integer number of volleys to be fired per mission.

Equation (4a) may be modified if either or both t_p and t_w are large in comparison to the time between firings of volleys. The time between volley firings may be obtained simply by taking the inverse of the rate of fire of a particular weapon. Conceptually, this time is the time necessary for the weapon to "recover" from the effects of firing a round. It may be seen from equation (4a) that the time between any two missions is the sum of t_p, t_w, and one "recovery time". It is obvious though that the recovery time may run "concurrently" with $(t_p + t_w)$. Therefore, if $(t_p + t_w)$ is larger than the recovery time, the time between any two missions is just $(t_p + t_w)$, and equation (4a) becomes:

$$\bar{t}_E = \frac{m(Nv - 1)}{R} + (m-1)t_p + t_d + (m-1)t_w \qquad (6)$$

The assumption that exponential distributions may be used for DSWS exposure times and enemy response times is reasonable for the capabilities which may exist in the future. However, for current and near-future capabilities, somewhat larger times may be expected. In this case, a distribution which reflects longer times, such as a log normal distribution, may be more appropriate.

The probability of a "timely" enemy counterfire response, given by equation (5), is based in part on the distribution of T_E, the artillery battery exposure time. The distribution is scaled by the average exposure time $\bar{t}_E$, which is a function of an average of numbers of rounds fired. Because the analysis of this thesis focuses on the vulnerability of a particular battery in a position, it is not appropriate to define T_E in terms of averages. The exposure time then is no longer considered

as a random variable; rather, it is defined directly by equations (4a) or (6) and is denoted by t_E. t_{E_K} is then defined as the exposure time remaining immediately after the firing of the first of k volleys, $t_{E_{K-1}}$ is the remaining exposure time after the second of k volleys, and so on.

IV. PROBABILITY OF COUNTERFIRE

A. GENERAL

The methodologies presented in Chapters II and III provide a means for obtaining an expression for the probability of an artillery battery receiving counterfire based on probabilities of detection by WLS and on the probability that the counterfire response by the enemy is "timely". This probability is given by the product of events 2 and 8 of Table 1, or:

$$P(\text{timely enemy response}|\text{detection})\ P(\text{detection}) \tag{7}$$

Define P_i^m (i=1,...,m) to be the probability that counterfire is received as a result of the firing of the ith of m volleys fired in a position. From equation (7),

$$P_i^m = P(T_R < t_{E_{m-i+1}})\ P_d \tag{8}$$

The cumulative probability of receiving counterfire before the position is vacated, accounting for the first j of m volleys (j=1,...,m) is then given by:

$$P_{CF_j}^m = 1 - \prod_{i=1}^{j} (1 - P_i^m) \tag{9}$$

assuming independence between events.

A conceptual approach yields the same result. Consider a battery which has just occupied a new position and will remain in that position

to fire m volleys. Immediately after the first volley is fired, the probability that counterfire will be received before the position is vacated is:

$$P^m_{CF_1} = P(T_R < t_{E_m})\, P_d = P^m_1 \tag{10}$$

For the second volley firing, the probability that counterfire will be received as a result of only that firing is the product of P^m_2 and the probability that counterfire is not received as a result of the first volley, or

$$P^m_2\,(1 - P^m_{CF_1}) \tag{11}$$

The cumulative probability of counterfire after the second volley is then

$$P^m_{CF_2} = P^m_{CF_1} + P^m_2\,(1 - P^m_{CF_1}) \tag{12}$$

and the cumulative probability of receiving counterfire before the position is vacated, accounting for the firing of the first j of m volleys is given by

$$P^m_{CF_j} = P^m_{CF_{j-1}} + P^m_J\,(1 - P^m_{CF_{j-1}}) \tag{13}$$

which gives the same result as equation (9).

B. PROBABILITY OF NOT RECEIVING COUNTERFIRE

The probability that counterfire is not received as a result of those firings prior to volley j, $(1-P^m_{CF_{j-1}})$, bears additional analysis. In a conceptual sense, counterfire will not be received if a detection

does not occur, if T_R is too long, or both. The methodology presented thus far assumes that, once a detection occurs, a counterfire process begins with an associated response time T_R which starts at the time of detection and ends with the impact of rounds at the battery's position. Therefore, if more than one detection of the same battery is obtained by one or more WLS, more than one counterfire process will occur. It would then be possible for counterfire to be received, for example, as a result of a detection on the third volley, even if a detection occurred on the first volley, because T_R for the first volley detection may be longer than that for the third volley detection.

In light of the description of the counterfire element given in Chapter I, this situation may not always be realistic. The counterfire element could be expected to consolidate multiple detections by WLS of a single firing battery into one counterfire mission. Consequently, only one counterfire process would occur with only one response time T_R. Since there is only one response time to be considered in this case, the probability of not receiving counterfire as a result of previous firings is simply the probability that no previous volleys were detected, or

$$(1 - P^{m}_{CF_{j-1}}) = (1 - P_d)^{j-1} \tag{14}$$

Equation (13) could then be rewritten as

$$P^{m}_{CF_j} = P^{m}_{CF_{j-1}} + P(T_R < t_{E_j})\, P_d\, (1 - p_d)^{j-1} \tag{15}$$

Note that the second term is simply the product of a timely response and the probability that the first detection of the battery occurs on

volley j. This probability then has a geometric distribution with parameter p_d. The cumulative distribution function gives the probability that a detection has occurred by volley j and will be denoted by P_{d_G}.

There are cases, however, where multiple counterfire processes may occur and equation (15) would not hold. These would include instances in which WLS are attached directly to artillery units as mentioned in Chapter I. It would be likely then that the counterfire element would not be involved in the consolidation of multiple detections by all WLS. Multiple processes may also occur in automated counterfire systems in which WLS-generated counterfire targets are transmitted directly to firing units. The effect would be, as in the previous example, that the counterfire element is effectively eliminated from the process, thereby eliminating the possibility of the consolidation of multiple detections of a battery into a single counterfire process.

The use of equation (13) with no modifications would account for all possibilities of multiple counterfire processes occurring. Using equation (13) modified as equation (15) would account for no such possibilities. Use of both would provide bounds for the problem since it could reasonably be expected that at least some degree of multiple processing could occur. For the illustrative example given in the next section then, probabilities of counterfire will be obtained both from equation (13) with no modifications ($P^m_{CF_j}$) and equation (13) modified as equation (15) ($\underline{P}^m_{CF_j}$).

C. ILLUSTRATIVE EXAMPLE

1. Scenario

A generic counterfire threat provides a suitable scenario for analysis of the application of the methodology. Table 2 lists values for specific hostile (Red) WLS parameters and distribution assumptions and for the friendly (Blue) artillery battery parameters. These values and assumptions are rather arbitrary in nature, yet clearly any ones of interest could be used. Values for m, the number of volleys to be fired from the position (and which defines the exposure time in the position), are taken to be 3 (Case 1), 5 (Case 2), and 7 (Case 3). Tables 3, 4, and 5 list the probabilities of counterfire for each volley, and Figures 3, 4, and 5 depict these probabilities graphically. (In this example, equation (6) is used rather than equation (4a).)

Table 2. Illustrative Example:
Red WLS and Blue Battery Parameters

Red WLS Parameters	Blue Battery Parameters
T_R = random variable of Red response times. Distribution is Rayleigh with parameters: t_L = location parameter = 5 minutes t = scale parameter = 10 minutes W = 10 km r = 10 km WLS #1: w=800 mils; P_c=.6; P_o=.5 WLS #2: w=1600 mils; P_c=.5; P_o=.9 (from equation (3), P_d=.582)	m = 5, 10, 15 M = 18 n = 6 R = 2 t_p = 15 seconds t_d = 5 minutes t_w = 2 minutes

Table 3. Case 1: m=3

VOLLEY	t_{E_j}	$P(T_R<t_{E_j})$	P_{d_G}	$\underline{P}^m_{CF_j}$	$P^m_{CF_j}$
1	12.5	.245	.582	.143	.143
2	12.0	.217	.825	.196	.251
3	11.5	.190	.927	.215	.334
4	9.25	.086	.970	.219	.367
5	8.75	.068	.988	.220	.392
6	8.25	.051	.995	.220	.410
7	6.0	.005	.998	.220	.412
8	5.5	.001	.999	.220	.412
9	5.0	.000	.999	.220	.412

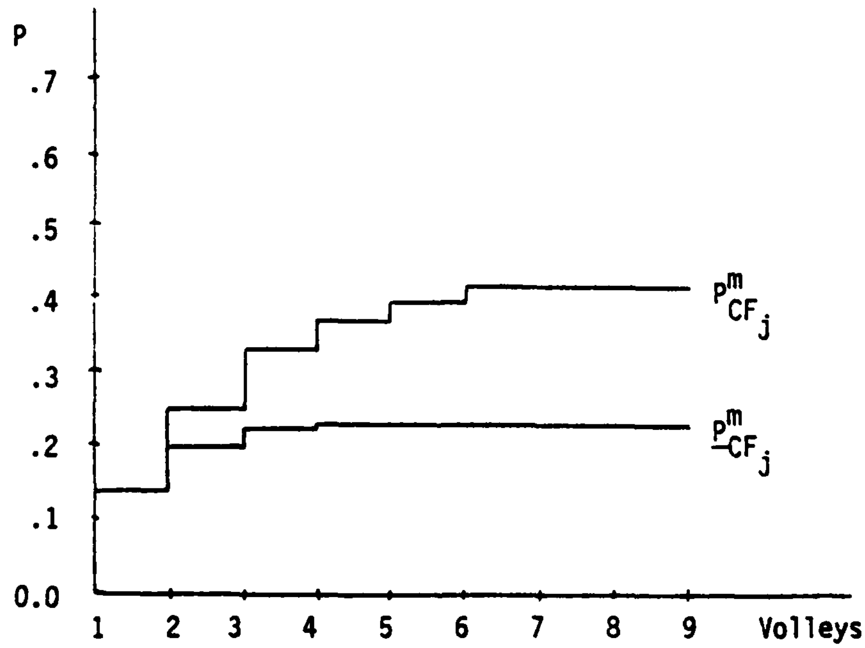

Figure 3. Case 1: m=3

Table 4. Case 2: m=5

VOLLEY	t_{E_j}	$P(T_R<t_{E_j})$	P_{d_G}	$\underline{P}^m_{CF_j}$	$P^m_{CF_j}$
1	19.0	.625	.582	.364	.364
2	18.5	.598	.825	.509	.585
3	18.0	.570	.927	.567	.723
4	15.75	.439	.970	.586	.794
5	15.25	.409	.988	.593	.843
6	14.75	.378	.995	.596	.877
7	12.5	.245	.998	.597	.894
8	12.0	.217	.999	.597	.907
9	11.5	.190	.999	.597	.917
10	9.25	.086	.999	.597	.921
11	8.75	.068	.999	.597	.924
12	8.25	.051	.999	.597	.926
13	6.0	.005	.999	.597	.926
14	5.5	.001	.999	.597	.926
15	5.0	.000	.999	.597	.926

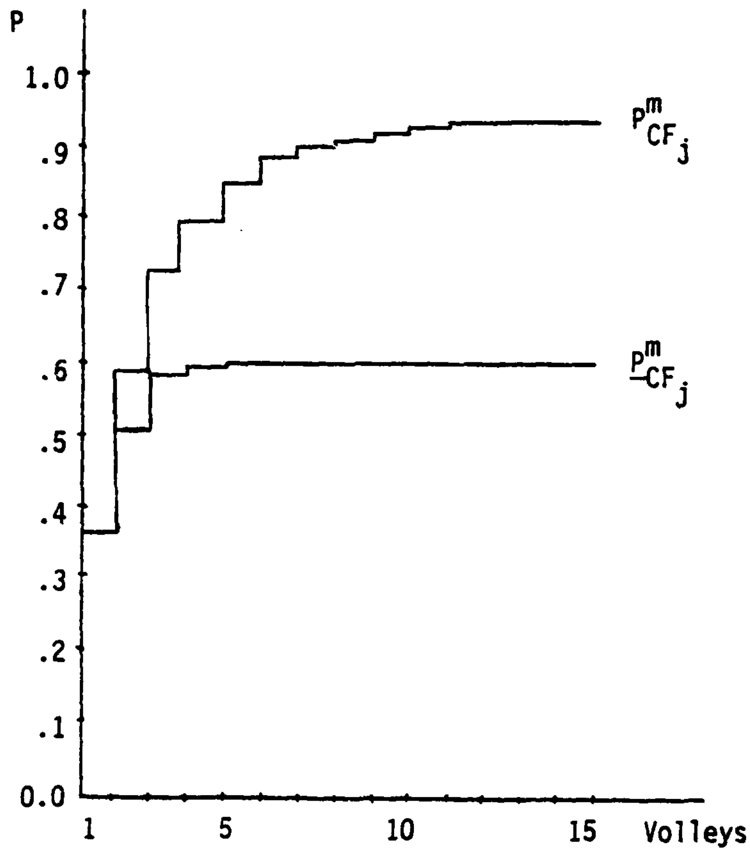

Figure 4. Case 2: m=5

Table 5. Case 3: m=7

VOLLEY	t_{E_j}	$P(T_R<t_{E_j})$	P_{d_G}	$\underline{P}^m_{CF_j}$	$P^m_{CF_j}$
1	25.5	.878	.582	.511	.511
2	25.0	.865	.825	.721	.757
3	24.5	.851	.927	.808	.877
4	22.25	.774	.970	.841	.932
5	21.75	.754	.988	.855	.962
6	21.25	.733	.995	.860	.978
7	19.0	.625	.998	.862	.986
8	18.5	.598	.999	.863	.991
9	18.0	.570	.999	.863	.994
10	15.75	.439	.999	.863	.995
11	15.25	.409	.999	.863	.995
12	14.75	.378	.999	.863	.995
13	12.5	.245	.999	.863	.995
14	12.0	.217	.999	.863	.995
15	11.5	.190	.999	.863	.995
16	9.25	.086	.999	.863	.995
17	8.75	.068	.999	.863	.995
18	8.25	.051	.999	.863	.995
19	6.0	.005	.999	.863	.995
20	5.5	.001	.999	.863	.995
21	5.0	.000	.999	.863	.995

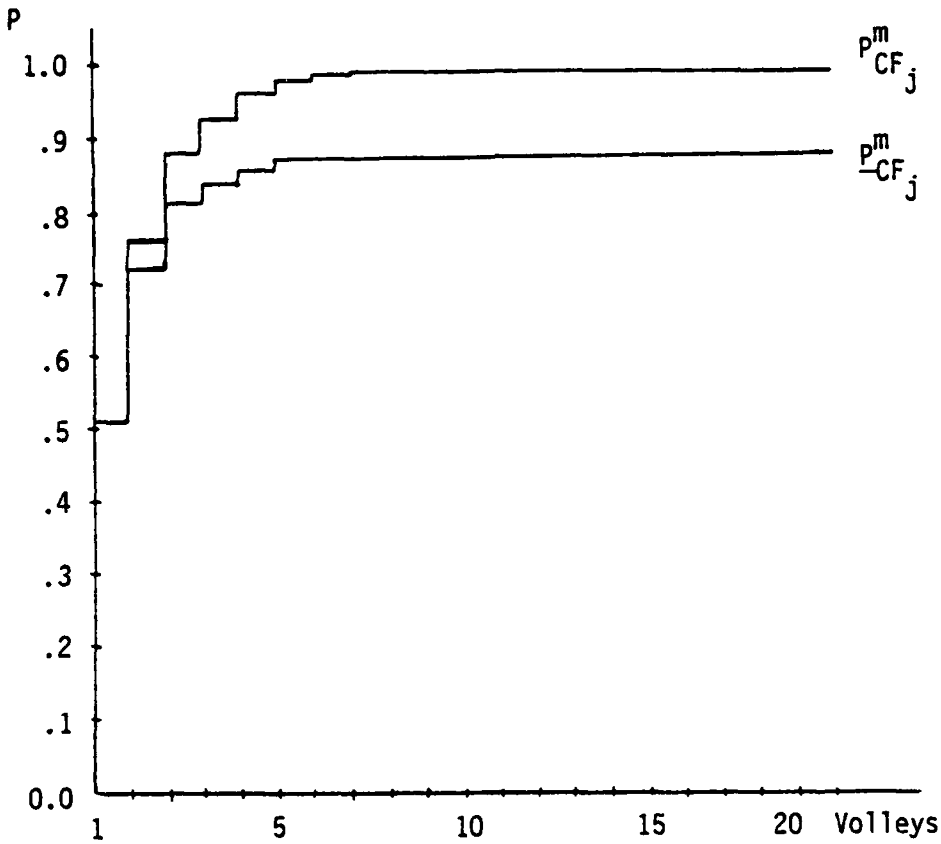

Figure 5. Case 3: m=7

2. <u>Discussion of Results</u>

As expected, $P^m_{CF_j}$ is never less than $\underline{P}^m_{CF_j}$ because of the additional probability resulting from multiple counterfire processes. $\underline{P}^m_{CF_j}$ reaches its maximum value more quickly than $P^m_{CF_j}$ because of the geometric properties of equation (15).

Differences between $P^m_{CF_j}$ and $\underline{P}^m_{CF_j}$ are much more pronounced in Case 2 than in Cases 1 and 3. In Case 1, $P(T_R<t_{E_j})$ is always small, so it would be expected that $P^m_{CF_j}$ and $\underline{P}^m_{CF_j}$ would not differ greatly. In Case 3, $P(T_R<t_{E_j})$ is still large when P_{d_G} reaches its maximum, which results in $\underline{P}^m_{CF_j}$ being large. For Case 2, however, $P(T_R<t_{E_j})$ is not so large when P_{d_G} reaches its maximum. Thus, $\underline{P}^m_{CF_j}$ reaches its maximum earlier than $P^m_{CF_j}$. The increases in $P^m_{CF_j}$ after this point again reflect the contribution of the additional probability of counterfire due to multiple counterfire processes.

3. <u>Parametric Variations</u>

To examine the impact of various tactical and equipment characteristic alternatives on the probability of counterfire, the model was exercised several more times for $m=5$ while varying appropriate Blue artillery battery parameters. Table 6 shows the results obtained by reducing the number of rounds fired from eighteen to twelve. Table 7 shows the impact of both decreasing the number of rounds fired and increasing the rate of fire from two to three rounds per minute. Table 8 lists the results for a decrease in waiting time between missions from two minutes to one minute. The results of these variations on the probability of counterfire are shown graphically in Figure 6. For these variations, only values for $P^m_{CF_j}$ are listed. Values for $\underline{P}^m_{CF_j}$ would obviously be lower as in previous cases.

Table 6. Case 4: m=5, M=12

VOLLEY	t_{E_j}	$P(T_R<t_{E_j})$	$P^m_{CF_j}$
1	16.5	.484	.282
2	16.0	.454	.472
3	13.75	.318	.570
4	13.25	.288	.642
5	11.0	.165	.676
6	10.5	.140	.702
7	8.25	.051	.711
8	7.75	.025	.715
9	5.5	.001	.715
10	5.0	.000	.715

Table 7. Case 5: m=5, M=12, R=3

VOLLEY	t_{E_j}	$P(T_R<t_{E_j})$	$P^m_{CF_j}$
1	15.67	.434	.253
2	15.33	.413	.433
3	13.08	.278	.525
4	12.74	.259	.597
5	10.5	.140	.630
6	10.16	.125	.657
7	7.91	.041	.665
8	7.57	.032	.671
9	5.33	.001	.671
10	5.0	.000	.671

Table 8. Case 6: $m=5$, $T_w=1$

VOLLEY	t_{E_j}	$P(T_R<t_{E_j})$	$P^m_{CF_j}$
1	15.0	.393	.229
2	14.5	.363	.392
3	14.0	.333	.510
4	12.75	.259	.584
5	12.25	.231	.640
6	11.75	.204	.683
7	10.5	.140	.709
8	10.0	.117	.729
9	9.5	.096	.744
10	8.25	.051	.752
11	7.75	.037	.757
12	7.25	.025	.761
13	6.0	.005	.762
14	5.5	.001	.762
15	5.0	.000	.762

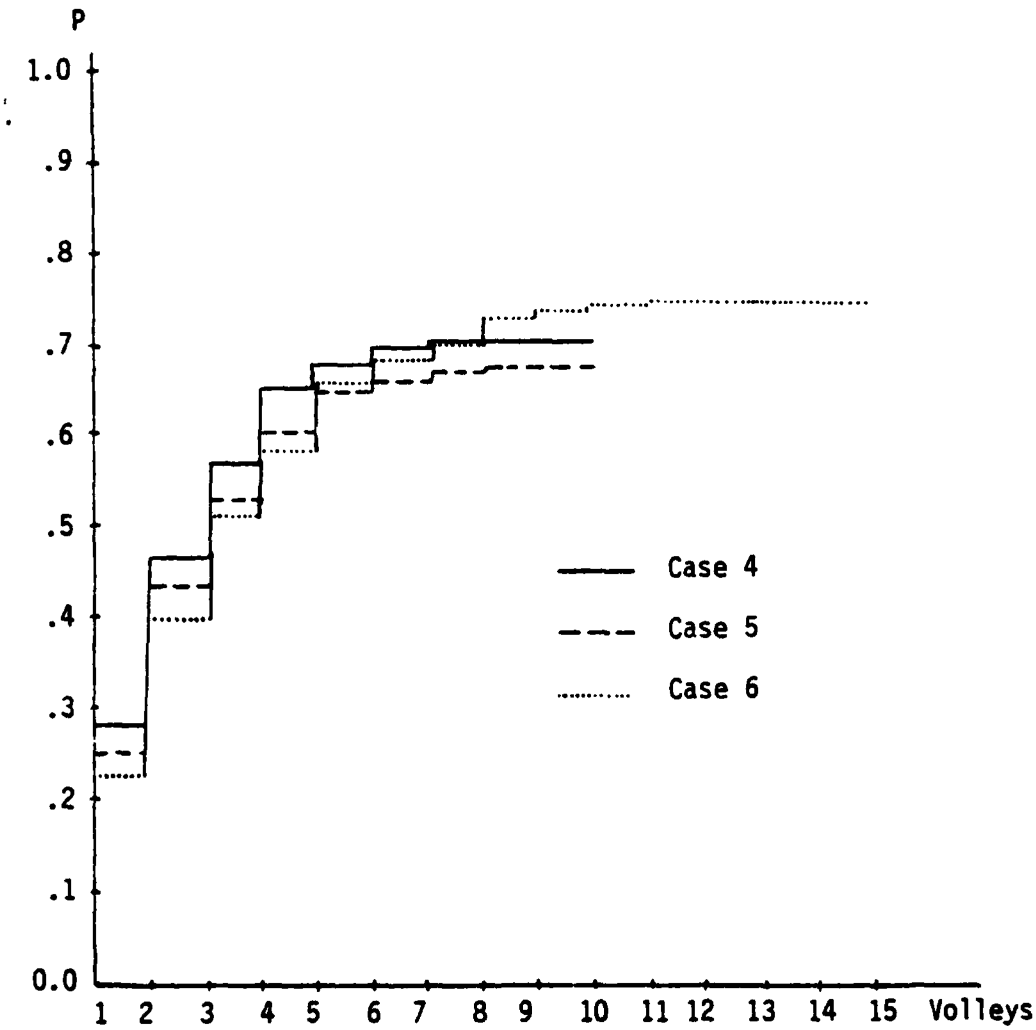

Figure 6. Parametric Variations: Cases 4, 5, and 6

For Case 4 (m=5, M=12), the probability of counterfire has decreased significantly from Case 2. A slightly larger reduction is achieved in Case 5 (m=5, M=12, R=3), though the reduction may not be significant relative to Case 4. The results of Case 6 (m=6, T_w=1), which could be viewed as an increase in the intensity of combat or as a decrease in technical fire direction time, show less improvement than Cases 5 or 5, even though the exposure time is less than those of Cases 4 and 5. This is due of course to the larger number of volleys of Case 6.

An additional variation was performed to examine the results of an increase in t_L and t, the parameters of the distribution of enemy response times. The results of this case are shown in Table 9 and in Figure 7. It can be seen that the probability of counterfire has decreased significantly from Case 2, and the results suggest the benefit in terms of survivability which may be derived by affecting the enemy's counterfire process in terms of increased response times.

These results show quite clearly that, for this model, a fairly large reduction in exposure time is necessary to reduce by a significant amount the probability of counterfire. This points out the potential worth in terms of survivability of highly mobile artillery weapons with high rates of fire and automated fire control systems.

Table 9. Case 7: $m=5$, $t_L=8$, $t=13$

VOLLEY	t_{E_j}	$P(T_R<t_{E_j})$	$P^m_{CF_j}$
1	19.0	.511	.297
2	18.5	.479	.493
3	18.0	.447	.625
4	15.75	.299	.690
5	15.25	.267	.738
6	14.75	.236	.774
7	12.5	.113	.789
8	12.0	.090	.800
9	11.5	.070	.808
10	9.25	.009	.809
11	8.75	.003	.809
12	8.25	.000	.809
13	6.0	.000	.809
14	5.5	.000	.809
15	5.0	.000	.809

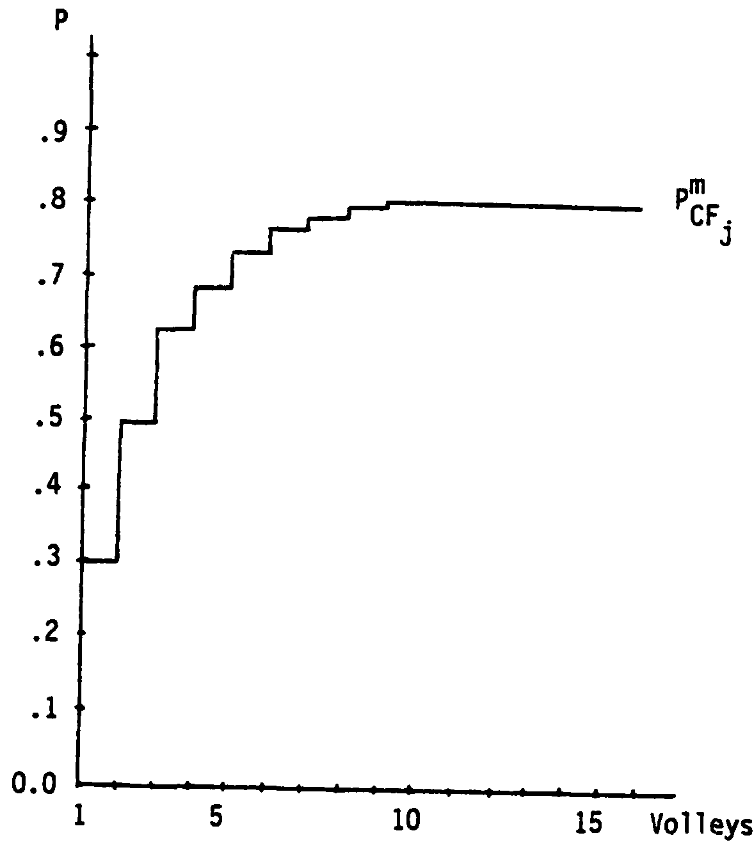

Figure 7. Case 7: m=5, t_L=8, t=13

V. FINAL COMMENTS

A. GENERAL

The methodology developed in the preceding chapters provides an analytical approach for the investigation of the vulnerability of a field artillery battery to counterfire in a given situation. It should be recognized, however, that this is only a partial analysis of the issue. A more complete systems approach would attempt to describe the account for the other events in the counterfire process (from Table 1) which were not addressed.

B. COMMENTS ON THE METHODOLOGY

Several inherent assumptions exist within the structure of the model which have not been, and should be, addressed explicitly. These tend to make the model rather situation-specific, and a wider application may require some modification.

The structure of equation (3) implies that a linear or near-linear forward edge of the battle area (FEBA) exists. This expression should be altered appropriately for a situation where a nonlinear FEBA would exist, such as in a encirclement.

It is also implicitly assumed that P_s is an appropriate expression for sound ranging systems. In reality, both P_s and P_c, the conditional probability of detection, are dependent on the configuration of the layout of the microphone base. The area of coverage of a sound ranging system is decreased by the employment of a curved microphone base, and is increased by the use of a linear base.

The model also assumes that weapons fire simultaneously in a volley. Thus, only one detection can possibly occur for a volley of several rounds.

C. AREAS FOR FURTHER ANALYSIS

It is safe to assert that vulnerability to counterfire will continue to be an issue of concern in the future. The extended ranges and high rates of fire of new artillery weapons, the increased lethality of munitions, and the increased accuracy of WLS require that artillery tactics and doctrine be constantly reviewed and modified if necessary to keep pace with these technological advances in the counterfire arena.

As mentioned in Chapter I, WLS are only one means by which an artillery battery may be located. The contributions of all intelligence sources should be considered for a complete vulnerability analysis. The contribution to the counterfire effort of communications direction finding systems would be of special interest because of the large numbers of these systems in the forces of the Soviet Union.

An analysis of the Soviet counterfire system would also be of interest. It is known that Soviet commanders would place a high priority on the destruction of U.S. nuclear-delivery means [Ref. 6]. An assessment of the effects of an intense, rapidly moving conflict on the Soviets' ability to wage a successful counterfire campaign with current and projected capabilities would be of significant value.

Numerous weapons effects models which describe the casualties inflicted on various types of targets by artillery fire are in existence. An analysis which addressed the counterfire issue using one of these

types of models in conjunction with an approach of the type presented in this thesis would give a more complete assessment of counterfire effectiveness in terms of probabilities of detection, timely enemy response, and weapons effects.

A final area of interest would be the applicability of this methodology to an assessment of the vulnerability of mortar units to counterfire. The similarities of artillery and mortar units are such that the methodology may be appropriate; however, some modifications may be necessary.

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